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On the critical behaviour of bosonised models in a long-range correlated random field

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Abstract. The renormalisation group approach is used for exploring the critical behaviour of bosonised models in the presence of a long-range correlated random field. The zero-temperature regime is studied in some detail for different types of random field correlation functions. As a result, the random critical exponents are not derivable from the pure ones with appropriate dimensional shifts.

1. Introduction

Quite recently (Aharony *et al* 1982, Busiello *et al* 1983a, 1984a, Uzunov *et al* 1984) a number of results have been obtained about the effects of a random quenched field on the critical properties of quantum systems, with and without the use of the replica trick (Edwards and Anderson 1975, Emery 1975). When a short-range correlated random field is concerned (Aharony *et al* 1982, Busiello *et al* 1983a), the quantum fluctuations, just like thermal ones, appear to be irrelevant relative to those caused by the randomness. Furthermore, for models such as the transverse Ising model at zero temperature, the switch of a longitudinal random field generates a dimensional cross-over $d \to d-3$ for which the random critical exponents are obtainable from the pure ones with d replaced by $d-3$ (Aharony *et al* 1982). In contrast, for bosonised models (Bose system, XY model in a transverse field, spin $-\frac{1}{2}$ planar ferromagnet, etc) (De Cesare 1978, Busiello and De Cesare 1980a, b, Uzunov 1981, Busiello *et al* 1983b) a dimensional shift $d \to d-4$ occurs but this cannot be interpreted as a dimensional crossover (Busiello *et al* 1983a). This situation has been confirmed by a Hartree study for an n-vector Bose model (Busiello *et al* 1984a) and a renormalisation group (RG) treatment (Uzunov *et al* 1984) for non-bosonised systems (structural phase transitions, transverse Ising model, etc) (Hertz 1976, Busiello *et al* 1983b) in the presence of a long-range correlated random field of the type already considered for the ideal Bose gas (Lacour-Gayet and Toulouse 1974) and for classical systems (Kardar *et al* 1983, Chang and Abrahams 1984). Nevertheless, in this case conditions are found under which the thermal and quantum fluctuations are dominant relative to the random field ones.

This paper is devoted to an RG investigation of the critical behaviour of bosonised models in the presence of a long-range correlated random field. Our purpose is three-fold:

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(i) to generalise the RG study of Busiello *et al* (1983a) to a Bose system with a free particle energy spectrum and a random field correlation function which behave like k^σ $(0<\sigma\leqslant 2)$ and $\Delta_{01}+\Delta_{02}k^\theta$ (arbitrary θ) in the wavevector space respectively;

(ii) to complement the self-consistent results of Busiello *et al* (1984a); and

(iii) to integrate the RG analysis made by Uzunov *et al* (1984) for non-bosonised models.

2. The behaviour of bosonised models

Here we use the Bose gas terminology but the results are true for other bosonised models apart from a different meaning of the coupling parameters (De Cesare 1978, Busiello and De Cesare 1980a, b, Uzunov 1981, Busiello *et al* 1983b). We consider a d-dimensional $(n/2)$-component Bose model in the presence of a quenched random field $\boldsymbol{h}(\boldsymbol{x})\equiv\{h^j(x); j=1,\ldots,n/2\}$, described in terms of the functional representation (Busiello *et al* 1984a):

$$\mathscr{Z}\{h\}=\int \mathscr{D}[\psi]\exp(-\mathscr{H}\{\psi,h\}) \tag{1}$$

where, for a given configuration of the random field, $\mathscr{Z}(h)$ is the grand partition function and the action $\mathscr{H}\{\psi,h\}$ is given by

$$\begin{aligned}\mathscr{H}\{\psi,h\}=&\sum_{j=1}^{n/2}\sum_{\substack{q\\0<|\boldsymbol{k}|<\Lambda}}(r_0+ck^\sigma-\mathrm{i}\omega_l)\psi^{j*}(q)\psi^j(q)\\&+\frac{u_0T}{4V}\sum_{i,j=1}^{n/2}\sum_{\substack{\{q_\nu\}\\0<|\boldsymbol{k}_\nu|<\Lambda}}\psi^{i*}(q_1)\psi^{j*}(q_2)\psi^i(q_3)\psi^j(q_1+q_2-q_3)\\&+\sum_{j=1}^{n/2}\sum_{\substack{q\\0<|\boldsymbol{k}|<\Lambda}}T^{-1/2}\delta_{\omega_l,0}(h^j_{\boldsymbol{k}}\psi^{j*}(q)+h^{j*}_{\boldsymbol{k}}\psi^j(q)).\end{aligned} \tag{2}$$

In (2) $\boldsymbol{\psi}\equiv\{\psi^j; j=1,\ldots,n/2\}$ is a complex field, $q\equiv(\boldsymbol{k},\omega_l)$, $\boldsymbol{k}$ is a wavevector for which a cut-off Λ is assumed, $\omega_l=2\pi Tl$ $(l=0,\pm1,\pm2,\ldots)$ are the Matsubara frequencies, T is the temperature, $-r_0=\mu$ is the chemical potential, $h^j_{\boldsymbol{k}}=V^{-1/2}\int \mathrm{d}^dx\, h^j(\boldsymbol{x})\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x})$ are the Fourier components of the random field and V is the volume of the systems. We shall assume that $\{h^j_{\boldsymbol{k}}\}$ are Gaussian random variables with averages

$$\begin{aligned}&(h^{j*}_{\boldsymbol{k}})_{\mathrm{av}}=(h^j_{\boldsymbol{k}})_{\mathrm{av}}=0\\&(h^{i*}_{\boldsymbol{k}}h^j_{\boldsymbol{k}'})_{\mathrm{av}}=\delta_{ij}\delta_{\boldsymbol{k}\boldsymbol{k}'}g(\boldsymbol{k})\end{aligned} \tag{3}$$

where the random correlation function $g(\boldsymbol{k})$ has the asymptotical form (Uzunov *et al* 1984, Kardar *et al* 1983, Chang and Abrahams 1984)

$$g(\boldsymbol{k})\approx\Delta_{01}+\Delta_{02}k^\theta \qquad \text{(arbitrary } \theta) \tag{4}$$

for small k. Of course $g(\boldsymbol{k})$ must be non-negative for all wavevectors up to the cut-off since it is the Fourier transform of the translationally invariant correlation function

$\bar{g}(x-y) = (h^{i*}(x)h^{i}(y))_{av}$. This imposes some restrictions on the possible values of the parameters Δ_{0i} $(i=1,2)$.

The standard RG techniques for random systems, making use of the replica trick (Busiello *et al* 1983a, 1984a) or the direct approach (Aharony *et al* 1982, Uzunov *et al* 1984) can now be applied to the functional representation (1) and (2). We find that the T-dependent RG differential equations to second order in the appropriate coupling parameters (Busiello *et al* 1983a) r, u, $w_i = u\Delta_i$ $(i=1,2)$ are (with units in which $c=\Lambda=1$)

$$\frac{\mathrm{d}r}{\mathrm{d}l} = \sigma r + k_d \frac{n+2}{4}\left(uA(r,T) + \frac{w_1+w_2}{(1+r)^2}\right)$$

$$\frac{\mathrm{d}u}{\mathrm{d}l} = (\sigma - d)u - \frac{k_d}{4}\left(u^2[(n+6)B(r,T)+2C(r,T)] + 2(n+8)\frac{u(w_1+w_2)}{(1+r)^3}\right)$$

$$\frac{\mathrm{d}w_1}{\mathrm{d}l} = (3\sigma - d)w_1 - \frac{k_d}{4}\left(uw_1[(n+6)B(r,T)+2C(r,T)] + 2(n+8)\frac{w_1(w_1+w_2)}{(1+r)^3}\right) \tag{5}$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}l} = (3\sigma - \theta - d)w_2 - \frac{k_d}{4}\left(uw_2[(n+6)B(r,T)+2C(r,T)] + 2(n+8)\frac{w_2(w_1+w_2)}{(1+r)^3}\right)$$

$$\frac{\mathrm{d}T}{\mathrm{d}l} = \sigma T$$

where $k_d = 2^{1-d}\pi^{-d/2}/\Gamma(d/2)$ and

$$A(r,T) = \{\exp[(1+r)/T] - 1\}^{-1}$$
$$B(r,T) = \tfrac{1}{4}T^{-1}\sinh^{-2}[(1+r)/2T] \tag{6}$$
$$C(r,T) = \tfrac{1}{2}(1+r)^{-1}\coth[(1+r)/2T].$$

In (5) we have assumed $\eta = 2-\sigma$ and $z=\sigma$ for the exponents which enter the rescaling of the fields and the frequencies, respectively. Of course, the physical region is defined by $u \geq 0$, $w_1+w_2 \geq 0$, with $w_1 \geq 0$ if $\theta > 0$ and $w_2 \geq 0$ if $\theta < 0$. The case $\theta = 0$, which corresponds to a short-range correlated random field with $\Delta_{01}+\Delta_{02} = h_0^2$, has already been discussed by Busiello *et al* (1983a). This situation also occurs when $w_2 \equiv 0$ or when $\theta > 0$ with $w_i \neq 0$ $(i=1,2)$. We must note that $O(w_1^3, w_2^3, w_1^2 w_2, w_1 w_2^2)$ contributions to w_1 are not generated by the differential RG iterations. These, which are present in the finite RG recursion relations (Busiello *et al* 1983a), would involve two momentum integrations over infinitesimal shells of width δl and would therefore vanish in the limit $\delta l \to 0$. Thus, in the next order in the coupling parameters, the equation for w_1 does not contain isolated terms in w_2^3; thus $w_1 = 0$ $(\Delta_1 = 0)$ implies $\mathrm{d}w_1/\mathrm{d}l = 0$ at least up to this order. Therefore the present random problem is simplified by initially taking $\Delta_1 \equiv 0$. We expect that this peculiarity consistently persists at higher orders with the corresponding results in the large-n limit (Busiello *et al* 1984a) and in contrast with the case of long-range correlated quenched impurities which couple quadratically to the order parameter both for classical (Weinrib and Halperin 1983) and quantum systems (Busiello *et al* 1984b).

For $T \neq 0$, since $T(l) \to \infty$ by iteration of the RG transformation, in terms of the new parameter $v = uT$, equations (5) reduce to the equations for the corresponding classical n-vector model and the results of Uzunov *et al* (1984) and Kardar *et al* (1983) are readily reproduced.

Let us consider now the $T=0$ regime where the quantum fluctuations might play a role. Equations (5) reduce to

$$\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}l} &= \sigma r + k_d \frac{n+2}{4}\frac{w_1+w_2}{(1+r)^2} \\
\frac{\mathrm{d}u}{\mathrm{d}l} &= (\sigma-d)u - \frac{k_d}{4}\frac{u^2}{1+r} - \frac{k_d}{2}(n+8)\frac{u(w_1+w_2)}{(1+r)^3} \\
\frac{\mathrm{d}w_1}{\mathrm{d}l} &= (3\sigma-d)w_1 - \frac{k_d}{4}\frac{uw_1}{1+r} - \frac{k_d}{2}(n+8)\frac{w_1(w_1+w_2)}{(1+r)^3} \\
\frac{\mathrm{d}w_2}{\mathrm{d}l} &= (3\sigma-\theta-d)w_2 - \frac{k_d}{4}\frac{uw_2}{1+r} - \frac{k_d}{2}(n+8)\frac{w_2(w_1+w_2)}{(1+r)^3}.
\end{aligned} \tag{7}$$

Now we discuss separately the cases ($w_1 \equiv 0$, $w_2 \geqslant 0$) and ($w_1 \neq 0$, $w_2 \neq 0$).

2.1. $w_1 \equiv 0$, $w_2 \geqslant 0$

In this case equations (7) show a peculiar degeneracy for $\theta = 2\sigma$ which will be considered later. With $\theta \neq 2\sigma$ the fixed points can be determined from (7) with $w_1 \equiv 0$ and the three eigenvalues λ_r, λ_u, λ_{w_2}, for each fixed point can be obtained, as usual, by diagonalising the corresponding linear matrix from (7). They are listed in table 1 and table 2 respectively.

Table 1. Fixed points of the RG equations for $w_1 \equiv 0$, $w_2 \neq 0$.

Fixed points	r^*	u^*	w_2^*
I Gaussian	0	0	0
II Long-range disorder	$-\frac{1}{2\sigma}\frac{n+2}{n+8}(3\sigma-\theta-d)$	0	$\frac{2(3\sigma-\theta-d)}{k_d(n+8)}$
III Pure	0	$\frac{4}{k_d}(\sigma-d)$	0

Table 2. Eigenvalues for each fixed point of table 1.

Fixed points	λ_r	λ_u	λ_{w_2}
I Gaussian	σ	$\sigma-d$	$3\sigma-\theta-d$
II Long-range disorder	$\sigma-\frac{n+2}{n+8}(3\sigma-\theta-d)$	$\theta-2\sigma$	$d+\theta-3\sigma$
III Pure	σ	$d-\sigma$	$2\sigma-\theta$

The Gaussian fixed point (FP) (I) is stable for $d/\sigma > 1$ and $(d+\theta)/\sigma > 3$ and for these values of (d, σ, θ) the critical behaviour of the system will be Gaussian. In the case of the long-range FP (II), in order for it to be stable, it must be $(d+\theta)/\sigma < 3$ with $\theta/\sigma < 2$. For $(d+\theta)/\sigma < 3$ this FP is unphysical ($w_2^* < 0$) but in any case unstable. Thus, for $(d+\theta)/\sigma < 3$ and $\theta/\sigma < 2$, the critical behaviour of the system is dominated by the long-range randomness and for the correlation length exponent we have $\nu = \sigma^{-1} + \sigma^{-2}(n+2)(n+8)^{-1}(3\sigma - \theta - d)$ to first order in $\varepsilon_\theta = 3\sigma - \theta - d$. Finally, the pure FP (III) is stable for $d/\sigma < 1$ and $\theta > 2\sigma$, just where the long-range disorder FP is unstable. Therefore for $d/\sigma < 1$ and $\theta > 2\sigma$ the quantum fluctuations dominate the random ones and the presence of the random field has no effect on the pure critical behaviour in the quantum regime (De Cesare 1978, Busiello and De Cesare 1980a, b, Uzunov 1981). The regions in the $(d/\sigma, \theta/\sigma)$ plane, where the different FP are stable and the corresponding critical regimes for the system occur, are schematically shown in figure 1. The straight lines $(d/\sigma = 1, \theta > 2\sigma)$ and $((d+\theta)/\sigma = 3, d/\sigma > 1)$ separate the pure (III) and the random long-range (II) regimes from the Gaussian (I) one and here logarithmic corrections to Gaussian exponents are expected (local marginality, Pfeuty and Toulouse 1977).

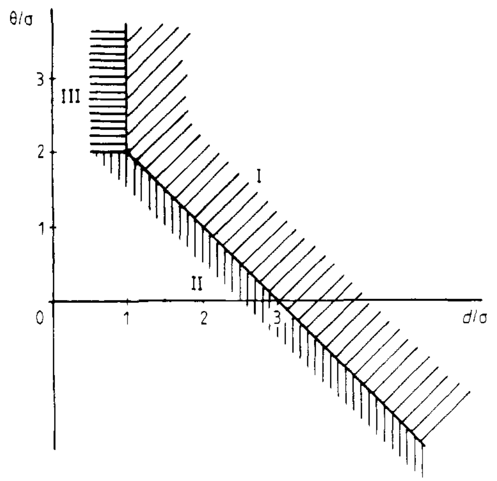

Figure 1. Stability regions for different fixed points when $w_1 = 0$, $w_2 \neq 0$.

We now investigate the case $\theta = 2\sigma$ which marks the borderline between II and III regimes for $d/\sigma < 1$ and constitutes a type of persistent marginality (Pfeuty and Toulouse 1977). In this case, the equations (7) show a degeneracy along the line of the parameter space:

$$(\sigma - d) - \frac{k_d}{4}[u + 2(n+8)w_2] = 0 \tag{8}$$

to $O(u, w_2)$.

For $d > \sigma$ the behaviour of the system is governed by the stable Gaussian FP. For $d < \sigma$, the degeneracy leads to departures from universality; the critical exponents correspond to the FP obtained as an intersection of a trajectory, specified by a set of initial conditions $(u_0, w_{02} = u_0\Delta_{02})$ and the line of degeneracy (8) (Pfeuty and Toulouse 1977, Tadic and Pirc 1984). Since from (7) we have $w_2 = u\Delta_{02}$, to first order in $(\sigma - d)$ the non-trivial FP for our random problem is

$$r^* = -\frac{1}{\sigma}\frac{(n+2)\Delta_{02}}{1+2(n+8)\Delta_{02}}(\sigma - d), \qquad u^* = \frac{4}{k_d}\frac{\sigma - d}{1+2(n+8)\Delta_{02}}$$
$$w_2^* = \frac{4}{k_d}\frac{\Delta_{02}(\sigma - d)}{1+2(n+8)\Delta_{02}} \tag{9}$$

with eigenvalues

$$\lambda_r = \sigma - \frac{2(n+2)\Delta_{02}(\sigma-d)}{1+2(n+8)\Delta_{02}} \qquad \lambda_u = -(\sigma-d) \qquad \lambda_{w_2} = -(\sigma-d). \tag{10}$$

The relations (10) show that for $d<\sigma$ the FP (9) is stable (for $d>\sigma$ is unphysical and unstable). For the correlation length exponent we find

$$\nu = \frac{1}{\sigma} + \frac{2}{\sigma^2}\frac{(n+2)\Delta_{02}(\sigma-d)}{1+2(n+8)\Delta_{02}} \tag{11}$$

and the other exponents can be valued in a standard way. Thus, for $\theta = 2\sigma$ and $d<\sigma$ the location of the stable FP and the values of the critical exponents are uniquely determined by the initial distribution of the impurities and, in this sense, we have a non-universal critical behaviour (Pfeuty and Toulouse 1977, Tadic and Pirc 1984). Of course, this result may be an artefact of the present approximation and to clarify this question one should have to include higher order terms in the recursion relations.

2.2. $w_1 \neq 0$, $w_2 \neq 0$

We must use the full equations (7) and the RG analysis is parallel to case 2.1 with the additional coordinate w_1 in the parameter space. Furthermore we assume $\theta \neq 0$ because, when $\theta = 0$, from (7) we obtain, as expected the RG equations for the parameters r, u, $w = w_1 + w_2$ appropriate for discussing the short-range disorder of the type considered in Busiello *et al* (1983a). The relevant FP of (7) and the corresponding eigenvalues are listed in tables 3 and 4 respectively.

Table 3. Fixed points of the RG equations for $w_1 \neq 0$, $w_2 \neq 0$.

Fixed points	r^*	u^*	w_1^*	w_2^*
I Gaussian	0	0	0	0
II Short-range disorder	$-\frac{1}{2\sigma}\frac{n+2}{n+8}(3\sigma-d)$	0	$\frac{2}{k_d(n+8)}(3\sigma-d)$	0
III Long-range disorder	$-\frac{1}{2\sigma}\frac{n+2}{n+8}(3\sigma-\theta-d)$	0	0	$\frac{2}{k_d(n+8)}(3\sigma-\theta-d)$
IV Pure	0	$\frac{4}{k_d}(\sigma-d)$	0	0

Table 4. Eigenvalues of each fixed point of table 3.

Fixed points	λ_r	λ_u	λ_{w_1}	λ_{w_2}
I Gaussian	σ	$\sigma-d$	$3\sigma-d$	$3\sigma-\theta-d$
II Short-range disorder	$\sigma-\frac{n+2}{n+8}(3\sigma-d)$	-2σ	$d-3\sigma$	$-\theta$
III Long-range disorder	$\sigma-\frac{n+2}{n+8}(3\sigma-\theta-d)$	$\theta-2\sigma$	θ	$d+\theta-3\sigma$
IV Pure	σ	$d-\sigma$	2σ	$2\sigma-\theta$

Table 4 shows that the FP I is stable for $d/\sigma > 3$ if $\theta > 0$ and for $(d+\theta)/\sigma > 3$ if $\theta < 0$. The FP II is stable only for $d/\sigma < 3$ and $\theta > 0$. For these values of d, θ and σ the critical behaviour is dominated, as expected, by the short-range disorder and we find $\nu = \sigma^{-1} + \sigma^{-2}(n+2)(n+8)^{-1}(3\sigma - d)$ to first order in $\varepsilon = 3\sigma - d$.

We note incidentally that when $\theta = 2\sigma$, we also have a degeneracy situation similar to that considered in 2.1 and the Δ_{02}-dependent FP (9) is present with the additional coordinate $w_1^* = 0$. Nevertheless, since in this case $\lambda_{w_1} = 2\sigma > 0$ this non-universal FP is unstable with respect to w_1 and is therefore irrelevant for our random problem.

In the case of the FP III, we see that it is stable only for $(d+\theta)/\sigma < 3$ and $\theta < 0$, where $\nu = \sigma^{-1} + \sigma^{-2}(n+2)(n+8)^{-1}(3\sigma - \theta - d)$ as in case 2.1 when the long-range disorder is relevant. Finally, since $\lambda_{w_1}^{(\mathrm{IV})} = 2\sigma > 0$, we have that, in contrast with case 2.1, the pure FP IV is in any case unstable, at least with respect to the short-range perturbation. This means that the quantum fluctuations are entirely dominated by the randomness. In figure 2 the different regions of stability in the $(d/\sigma, \theta/\sigma)$ plane for different FP are qualitatively shown.

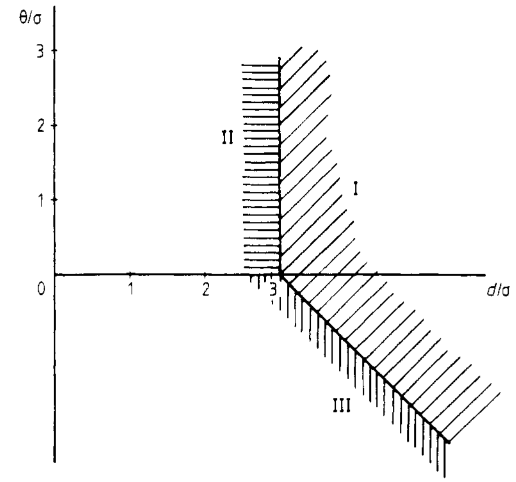

Figure 2. Stability regions for differerent fixed points when $w_1 \neq 0$, $w_2 \neq 0$.

3. Conclusion

In conclusion, the previous analysis indicates the following points.

(1) The results with $w_1 \equiv 0$ are consistent with the corresponding ones obtained by Busiello *et al* (1984a) by means of the Hartree approximation.

(2) When the disorder is relevant, the critical exponents are not derivable from the pure Gaussian ones (De Cesare 1978, Busiello and De Cesare 1980a, b, Uzunov 1981) with appropriate dimensional shifts. This confirms the results of Busiello *et al* (1983a) and it is peculiar to bosonised systems.

(3) Except for the case $w_1 \equiv 0$ with $d < \sigma$ and $\theta > 2\sigma$, the quantum fluctuations, as the thermal ones at $T \neq 0$, are irrelevant relative to those caused by the random field and the classical-quantum crossover in the low-temperature limit, which appears in the pure systems (De Cesare 1982) is destroyed.

(4) Even if the $(T=0)$-RG equations are different from those for non-bosonised systems, the random behaviour is similar to that discovered by Uzunov *et al* (1984).

References

Aharony A, Gefen Y and Shapir Y 1982 *J. Phys. C: Solid State Phys.* **15** 673
Busiello G and De Cesare L 1980a *Nuovo Cimento* B **59** 327
—— 1980b *Phys. Lett.* **77A** 177
Busiello G, De Cesare L and Rabuffo I 1983a *Phys. Rev.* B **28** 6463
—— 1983b *Physica* **117A** 445
—— 1984a *Physica* **126A** 195
—— 1984b *Phys. Lett.* **102A** 41
Chang M-C and Abrahams E 1984 *Phys. Rev.* B **29** 201
De Cesare L 1978 *Lett. Nuovo Cimento* **22** 325, 632
—— 1982 *Nuovo Cimento* D **1** 289
Edwards S F and Anderson P W 1975 *J. Phys. F: Met. Phys.* **5** 965
Emery V J 1975 *Phys. Rev.* B 11 239
Hertz J A 1976 *Phys. Rev.* B **14** 1165
Kardar M, McClain B and Taylor C 1983 *Phys. Rev.* B **27** 5875
Lacour-Gayet P and Toulous G 1974 *J. Physique* **35** 425
Pfeuty P and Toulouse G 1977 *Introduction to the Renormalization Group and to Critical Phenomena* (New York: Wiley)
Tadic B and Pirc R 1984 *J. Phys. C: Solid State Phys.* **17** 1023
Uzunov D I 1981 *Phys. Lett.* **87A** 11
Uzunov D I, Korutcheva E R and Millev Y T 1984 unpublished
Weinrib A and Halperin B I 1983 *Phys. Rev.* B **27** 413